The golden mean problem: we want to limit the available strategies to agents, but not to much so they will still be able to accomplish a variety of objectives.

$S-$a set of strategies available to agents, $G_{soc}-$a set of social objectives, $\forall agent_i : 1 \leq i \leq n, \quad \forall y \in G_{soc} : \qquad M_y^i : S^n \rightarrow [0,1]$ we are trying to find $\overline{S} \subseteq S$, where $\overline{S}-$a set of allowed strategies such that $\forall agent_i : 1 \leq i \leq n, \quad \forall g \in G_{soc}, \quad \exists s \in \overline{S} : \quad M_y^i(s, \sigma_{-i}) \geq \varepsilon, \quad \forall \sigma_{-i}^{n-1} \in S^{n-1}$.

the idea (in a two agent scenerio): $agent_1$has many was to acomplish goal $g_1 \in G_1$, but $agent_2$might hinder him. specificaly we need to limit the actions of $agent_2$, but because of simitry we also need to limit $agent_1$, and this might leave $agent_1$ no means to accomplish other goals.

assume $|S| = m < \infty, \quad n = 2, \quad goals : g_1, \ldots, g_k, \quad U_1(s,t) = U_2(t,s)$ $(utility\ functions\ are\ symetric$ (meaning utility return is non dependent of $agent_i$)

the utility function for goal g_l matches each $(s_i, s_j) \in S \times S$ a value in $[0,1]$ the matches the utility to $player_1$, where $player_1$plays $s_i$and $player_2$plays s_j.

It is possable to match, therfore for every g_l there exists a matrix M_l. given a sub-set $T \subseteq \{1, \ldots, m\}$, we define M_l^T to be a sub-matrix of M_l which we obtain by removing the lines and columns of T.

the golden mean problem becomes the problem find T such that for all M_l^Tcontains a line that every value in it is at least ε.

we now show that finding golden mean is NP-Complete.

in order to show that the problem is NP-Hard, it is enough to show that another problem that is knowen to be NP-Hard is effectively reducable to the golden mean problem. We will show a reduction form SAT.

SAT: there is a set of boolean variables $X = \{x_1, x_2, \ldots, x_n\}$, the CNF formula of these variables is $C_1 \cap C_2 \cap \ldots \cap C_m$ where for each $C_i : \ l_{i_1} \cup l_{i_2} \cup \ldots \cup l_{i_{m_2}}$ where $l_{i_s}$is $x_s$or $\neg x_s$for some $x_s \in X$.

given this CNF formula we ask if there exists an assignment that satisfies it.

so basicly we are trying to find out if exists x-s for which the CNF formula will give true.

lets take $\varepsilon = 1$(for Victor). For each cluase in the game we will match a matrix/goal(we showed earlier that they are equivalent). The $i'th$-line and the $i'th$-column in this matrix will match the variable x_i where $1 \leq i \leq n$, and the variable $\neg x_{i-n}$ where $n+1 \leq i \leq 2n$.

	x_1	x_2	$\ldots$	x_n	$\neg x_1$	$\neg x_2$	$\ldots$	$\neg x_n$
x_1	1	1	$1\ldots1$	1	0	1	$1\ldots1$	1
x_2	0	0	$0\ldots0$	0	0	0	$0\ldots0$	0
$\vdots$								
x_n								
$\neg x_1$								
$\neg x_2$	1	0	$1\ldots1$	1	1	1	$1\ldots1$	1
$\vdots$								
$\neg x_n$								

every entry of the type $(i, i+n)$ and $(i+n, i)$ where $x_1, \ldots, x_n$ are variables in the formula.

as for the rest of the entrys: in a certain line there will be 1, everywhere (except those entrys already assigned to be 0), If the desired literal apper in the clause, otherwise there will be 0.

now if exists a golden mean for each goal we will look for a strategy that garantes it, and we will collect these strategies. These strategies will match literals for which we will assign for this True, and for their negate False. We give some value for the remainder (note that this gives an assignment that satisfies)

if an assignment that satisfies exists, we will remove lines and columns that are matched with False, and we will remain with a set of strategies that asure a golden mean.

Definition(in game thoery): a Social Law is a restriction on the set of actions avaliable to the agents.

A game g and a social law sl induces a subgame $g_{sl}$of g, that is a restriction of g to the actions that are not prohibited by sl.

Let g be a game, V a game variable, and $<$ (smaller) an ordering on the possable value of V. A social law sl is the rational with respect to G and V if $V(g) < V(g_{sl})$

a social convention is a social law that limits to only one action, we will look at the development of social conventions

Definition: An $n-k-g$ iterative game consists of a set of $n \quad agents$, $k-person \quad game \quad g$, and an unbounded sequence of order tuples of $k-agents$ selected from a uniform distribution over the given agents.

Action selection function: a function from an $agent's$ history to an action in g, which is both oblivious(non dependent on the stratgy's name) and local(dependent only on the observed history (utilities and actions observed)).

HCR- Highest Cumulative Reward - an $agent$ switches to an action $\Longleftrightarrow$ the total payoff obtained from this action is the highest, and higher then his total payoff from the current action

	a	b
a	1,1	-1,-1
b	-1,-1	1,1

(schelling won a nobel prize for this analizing game),

	c	d
c	1,1	-3,3
d	3,-3	-2,-2

(a variation of the prisoner's dillema)

Theorem: Let $n \geq 4$ given an $n-2-g$ iterative game, where g is a social agreement game, placing no constraints on the inital choices then HCR(as a recuring/learning law) leads to rational convention with respect to maxmin("and other too") for any social agreement g. If HCR is applied with bounded memory, the memory can be selected so that the speed of convergence will asymtutically optimal.

skitch of proof:

(what we basicly are going to show is that given a game with n players, I tell them that if they play by this rule, they will converge on a rational convention, but even greater, I say that I can tell them how many iterations they need to remember so that the speed of convergenc will be $O(nlog(n))$ and that there exists no faster convergence then this.)

a more acurate definition of the requirement, $\forall \varepsilon > 0, \quad \exists m = m(\varepsilon)$ such that using HCR will lead to a convention with probability $1 - \varepsilon$ after m iterations, and will remain there.