Multi Agent Systems - Lecture 9

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Considerations

In this lecture Prof. Moshe Tennenholtz mainly exposed the concepts of *The Clarke Mechanism* and *Ex-post Equilibrium*. He used two sets of slides, firstly entitled as:

1. Asymptotically Optimal Multi-Object Auctions
2. Multi-Agent Systems

The Clarke Mechanism, when applied to an economic environment with many agents, allows matching the upper bound of the revenue-surplus inequality:

$$E(\tilde{R}) \leq E(\tilde{S}) \tag{1}$$

The inequality states that the maximal expected revenue of the center (seller) is never greater than the maximal expected surplus of the agents (buyers). The best way to resume the lecture will be mapping the slides that backgrounded the comments noted here. The resume starts with the subject called *The Clarke Mechanism*, in the first set of slides.

The Clarke Mechanism

Asymptotical Optimality We consider G a set of goods, and $g : M \to \Pi$ the *allocation function* that will allocate the goods in G to a subset of agents in N, depending on their messages (slide[18]).

The transfer function for agent i, $t_i(m)$, also known as the payment function, defines how much agent i must pay in order to obtain the subset of goods to it allocated. The **payment** can be calculated over the *reported valuations*, as the difference between the maximal sum obtained in an optimal allocation that ignores i, and the sum of reported valuations of all the agents except i, in the optimal allocation considering N.

The maximal revenue $\tilde{R}(v)$ approximates the maximal surplus $\tilde{S}(v)$ when the number of player increases (slide[19]). Considering a high number of players, we can proove that *The Clarke Mechanism* is asymptotically optimal (slide[21]), i.e., maximal revenue equals maximal surplus when $n \to \infty$.

Inequality Validation The conditions exposed in slide[22] are necessary to keep the revenue-surplus inequality [1] valid. A counter example can be seen in slide[23], where the valuation function w_i assigns a negative value for the allocation that supplies the one good being auctioned to the opponent agent: $w_i(3-i) = -3$. In this case, the expected revenue $E(\tilde{R}) = 2$ while the expected surplus $E(\tilde{S}) = 0$.

We can associate this example to a *patent licitation* case, in which an agent may not be willing to acquire the patent (has a low valuation for it), but would prefer buying it than letting it's opponent have it (negative valuation is given to the allocation that supplies the patent to the opponent).

In a risk-seeking agents' environment, the inequality neither holds, as shown in slide[25].

Ex-post Equilibrium

second set of slides

In a non-cooperative environment, where resources are to be allocated among several parties, two major obstacles should be avoided (slide[3]):

- parties may cheat about the worth of the different resources
- communicating the valuation for all possible allocations is infeasible

So what is a **rational behaviour** for *The Clarke Mechanism*, given the communication constraint (slide[6])? Assuming that agents play rational, we search for a strategy that will overcome the constraint, and yet characterize an equilibrium (slide[7]):

Strategy 1 *Agent i submits its valuation $v_i(G)$ for G, and 0 is considered as the value for $v_i(K)$, where K is any subset of G.*

The above strategy has very good communication complexity [1], but *terrible economic efficiency*. Two major challenges are thrown (slide[8]):

1. Characterize the **equilibria** in *The Clarke Mechanism*
2. Study the **tradeoff** between communication complexity and economic efficiency in equilibrium.

Environment and Mechanism We keep the same economic environment defined in the first set of slides, although different symbology is used (slide[9]). The possible valuation functions are known to all agents. The mechanism and the dynamics for resource allocation is also the one defined for *The Clarke Mechanism* (slide[10], slide[11]).

Strategy Profile Equilibrium A strategy for agent i is determined by the bid function $b_i : V_i \longrightarrow X_i$, where X_i is equivalent to M_i, the message set for agent i. A strategy profile b aggregates the strategies chosen by all agents $b = (b_1, b_2, ..., b_n)$ (slide[12]). b is in **ex-post equilibrium** if there is no strategy for any agent that will provide a better payoff, considering that the other agents will stick to their strategies chosen in the profile b.

[1] agent submits only one bid to the center

Example: In the board below the state space is defined as four possible games. *Nature* informs the `ROW` player about which row will be played, but not which one of the games. Still, there is a dominant strategy for player `ROW`: to choose `U` in the case of paying the top row, or to choose `D` in the case of playing the bottom row.

	L	R
U	$2,8$	$5,1$
D	$1,5$	$6,4$

	L	R
U	$2,8$	$5,1$
D	$1,5$	$6,4$

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